

Geometry-Aware Manifold Trajectory Modeling for Task fMRI Activation Mapping

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Abstract. Functional magnetic resonance imaging (fMRI) measures task-evoked neural activity through blood-oxygen-level-dependent (BOLD) signals and enables activation mapping to localize task-related brain responses. Conventional activation mapping typically relies on the general linear model (GLM), which assumes linearity and a predefined hemodynamic response function (HRF). However, mounting evidence indicates that BOLD responses are nonlinear and state-dependent. Meanwhile, the high dimensionality and low signal-to-noise ratio of fMRI further challenge the linear assumption. These limitations motivate a neural-manifold view, in which high dimensional task dynamics evolve on a low-dimensional state space. We propose a geometry-aware manifold trajectory modeling framework that represents task-related activity as structured trajectories. The framework identifies a central baseline cluster, computes geodesic distances from each embedded frame to this cluster along the manifold trajectory graph, and converts them into vertex-wise activation maps via the proposed Manifold Geodesic-based Similarity (MGS) without HRF fitting. Using the Human Connectome Project (HCP) motor-task fMRI dataset, we uncover a petal-like looped trajectory, indicating a reproducible low-dimensional organization of task-evoked dynamics. At the individual-level, MGS robustly recovers canonical motor activations despite substantial single-subject fMRI noise. At the group-level, compared with HRF-based GLM, MGS yields more discriminative activation patterns and highlights task-related effects less emphasized by GLM, demonstrating the advantage of geometry-aware manifold trajectories for task fMRI activation mapping. Code is available at https://github.com/PCMLYR/ManifoldTraj_MGS

Keywords: Brain manifold · fMRI · Geodesic distance · Brain activation mapping · General linear model.

1 Introduction

Functional Magnetic Resonance Imaging (fMRI) infers task-evoked brain activity through the blood-oxygen-level-dependent (BOLD) signal, whose contrast reflects changes in deoxyhemoglobin driven by neurovascular coupling [22]. A core

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objective of task fMRI is activation mapping: identifying brain regions whose BOLD time series are modulated by externally controlled cognitive states so that condition-specific functional organization can be localized and compared across subjects [1]. The prevailing analysis framework is the General Linear Model (GLM) [10], which assumes that task effects add linearly after convolution with a canonical hemodynamic response function (HRF) [4, 17]. While effective in many settings, GLM is sensitive to HRF mismatch and latency variability across regions and individuals [26]. More fundamentally, the transformation from neural activity to BOLD is not strictly linear and time-invariant: hemodynamic saturation, refractory interactions, and nonlinear vascular dynamics can violate the canonical HRF assumption and blur condition selectivity [11]. These limitations motivate activation measures that are less dependent on a fixed HRF model and can better accommodate nonlinear task-evoked dynamics [17].

A second bottleneck is that whole-brain BOLD data are intrinsically high dimensional and low signal-to-noise: each frame is a large activity vector, yet task-relevant structure is weak relative to physiological and measurement variability, making inference challenging under purely local modeling [8]. Task fMRI analyses can also be sensitive to analytic choices and may exhibit limited test-retest reliability in commonly used activation measures [3, 9]. These nonlinearities motivate a manifold viewpoint: task-evoked brain states may evolve on a low-dimensional nonlinear neural manifold as smooth trajectories that depart from and return to an intrinsic baseline [12, 24, 21, 25, 16]. A broad class of nonlinear embeddings has been used to recover such geometry from high-dimensional observations, including LLE [23], diffusion maps [7], t-SNE [18], UMAP [19], and PHATE [20]. Recent advances such as T-PHATE [6] explicitly exploit autocorrelative temporal structure to denoise fMRI time series and uncover interpretable state trajectories. Notably, these studies often reveal recurrent or loop-like trajectories that align with cognitive conditions [20, 6, 16]. However, these loops are frequently treated as qualitative signatures for visualization or segmentation, while their geometric meaning is rarely translated into a principled, geometry-aware activation statistic, particularly one that measures progression along the manifold (e.g., via geodesic distance) rather than in the ambient space.

Here we present a geometry-aware, data-driven framework for brain surface task fMRI activation mapping and demonstrate its utility on Human Connectome Project (HCP) motor-task fMRI dataset. Rather than relying on a fixed HRF model, we characterize task-evoked dynamics as structured state evolution on a low-dimensional manifold, where successive frames form a petal-like looped trajectory with condition-aligned excursions. This geometric view exposes an intrinsic task baseline region (dominated by fixation and quiescent transitions) and yields a temporally resolved distance-to-baseline signature that tracks task engagement. Leveraging this signature, we propose the Manifold Geodesic-based Similarity (MGS), which reliably recovers canonical motor somatotopy while producing more compact and condition-selective activation patterns than HRF-constrained GLM, and additionally highlights task-relevant effects that can be weak under purely linear, voxel-wise modeling.

In summary, our contributions are threefold: (1) We propose a geometry-aware manifold trajectory modeling framework that characterizes task fMRI as structured state evolution on a learned low-dimensional manifold. (2) We introduce a geodesic distance-to-baseline geometric measure on the learned manifold trajectory to provide a model-light reference for task engagement and subsequent activation inference. (3) Using HCP motor-task fMRI, we uncover reproducible petal-like looped manifold trajectories with a central baseline cluster and demonstrate that the resulting MGS-based maps recover canonical motor organization while yielding more compact and discriminative group-level patterns than conventional HRF-based GLM.

2 Materials and Methods

2.1 Dataset and Preprocessing

We analyzed WU-Minn Human Connectome Project Young Adult (HCP-YA) 2025 release [27] resting-state and motor task fMRI data from a subset of 100 unrelated healthy adults (22–35 years; sex-balanced), acquired on a 3T Siemens Connectome Skyra (TR=0.72s; 2 mm isotropic). Data were processed using the HCP minimal preprocessing pipeline [15] and the 2025 release denoising outputs (sICA+FIX with reclean and temporal ICA; spin-echo field-map assistance) [13, 14], yielding CIFTI grayordinate BOLD time series. We additionally applied 4-mm FWHM smoothing and vertex-wise mean-centering, without temporal filtering. The motor task followed the HCP design [5] with five cue-driven

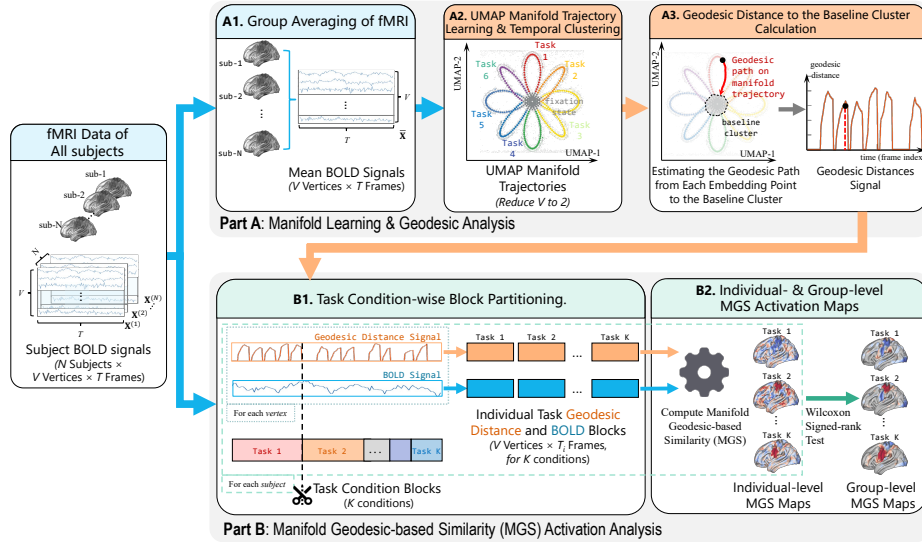


Fig. 1. Overview of the proposed manifold trajectory learning and MGS activation mapping framework.

movements (left/right hand, left/right foot, tongue). All subjects completed two phase-encoding runs (LR/RL), each with 10 task blocks (2 per condition) separated by 15-s fixation.

2.2 Manifold Learning and Geodesic Analysis

Group Averaging of fMRI. Let $\mathbf{X}^{(n)} \in \mathbb{R}^{V \times T}$ denote the preprocessed task fMRI time series of subject n ($n = 1, \dots, N$), where V is the number of brain surface vertices and T is the number of task frames. We first computed a group-averaged sequence $\bar{\mathbf{X}} = \frac{1}{N} \sum_{n=1}^N \mathbf{X}^{(n)} \in \mathbb{R}^{V \times T}$ as shown in Fig. 1(A1), and treated each frame t as a V -dimensional vector $\bar{\mathbf{x}}_t = \bar{\mathbf{X}}_{:,t} \in \mathbb{R}^V$.

UMAP Manifold Trajectory Learning and Temporal Clustering. The shared low-dimensional manifold representation was learned by mapping $\bar{\mathbf{x}}_t$ to $\mathbf{y}_t \in \mathbb{R}^2$ using UMAP [19] (task frames only; LR/RL concatenated; $n_{\text{neighbors}} = 40$; standardized Euclidean distance) as shown in Fig. 1(A2). The ordered set $\Gamma = \{\mathbf{y}_t\}_{t=1}^T$, with consecutive points connected by time index, defines a manifold trajectory capturing whole-brain state evolution during the task. This trajectory serves as the geometric substrate for subsequent distance-to-baseline computation.

We identified recurrent manifold states by clustering $\{\mathbf{y}_t\}$ using Density-based Merging (DBM) [28]. Among the resulting clusters, we defined the task baseline cluster C_{base} as the union of the one or two DBM clusters whose centroids are closest to the embedding center; if only one such cluster exists, we use that cluster alone. C_{base} is the central, high-density region of the embedding dominated by fixation and run-onset quiescent frames.

Geodesic Distance to the Baseline Cluster Calculation. We represented the ordered embeddings $\{\mathbf{y}_t\}_{t=1}^T$ as a trajectory graph on the learned manifold by connecting consecutive frames in time, and the geodesic was approximated by temporal adjacency. The resulting trajectory graph has edge weights $\ell_t = \|\mathbf{y}_{t+1} - \mathbf{y}_t\|_2$ between adjacent nodes. Specifically, for each frame t , we quantify the deviation of its temporal embedding point from the within-task baseline (baseline cluster C_{base}) by the shortest path from this point to the baseline cluster on the trajectory graph, i.e., the geodesic distance to baseline, defined as:

$$d_G(t) = \min_{t' \in C_{\text{base}}} \text{dist}(t, t'), \quad (1)$$

where $\text{dist}(t, t')$ is the shortest-path distance on the undirected trajectory graph from temporal embedding point t to t' (can be traversed forward or backward in time), so that each frame is assigned its own along-trajectory geodesic path and distance to the baseline cluster. We computed $d_G(t)$ on the manifold trajectory learned from the group-averaged fMRI and reused it for all subjects

to obtain a stable, shared geometric feature of task-evoked dynamics as illustrated in Fig. 1(A3). Thus, individual-level variability was captured only through vertex-wise BOLD blocks in later steps.

2.3 Manifold Geodesic-based Similarity (MGS) Activation Analysis

Task Condition-wise Block Partitioning. To align task events with hemodynamic responses, we used a fixed shift of 6 frames (≈ 4.32 s for the HCP dataset) as a coarse latency alignment rather than HRF fitting. Using the delay-shifted labels, each subject’s vertex-wise time series was segmented into condition blocks, denoted $\mathbf{f}_{v,k,b}^{(n)}$ for vertex v , condition k , and repetition b as shown in Fig. 1(B1). Using the same time windows, we extracted the matched geodesic distance-to-baseline signal $\mathbf{d}_{G,k,b}$ calculated by Eq.(1).

Individual- and Group-Level MGS Activation Maps Calculation. We define the Manifold Geodesic-based Similarity (MGS) as the alignment between a BOLD block and its corresponding manifold-distance pattern. As Fig. 1(B2) shows, for each block, we mapped distances to a bounded weight vector $w(\mathbf{x}) = 1 - \exp(-\mathbf{x})$ (element-wise) and computed a block score by the inner product between the ℓ_2 -normalized BOLD block and $w(\mathbf{d}_{G,k,b})$. Averaging over the B_k repetitions of condition k gives

$$\text{MGS}_{v,k}^{(n)} = \frac{1}{B_k} \sum_{b=1}^{B_k} \left\langle \frac{\mathbf{f}_{v,k,b}^{(n)}}{\|\mathbf{f}_{v,k,b}^{(n)}\|_2 + \varepsilon}, w(\mathbf{d}_{G,k,b}) \right\rangle, \quad (2)$$

where ε is a small constant for numerical stability. For each condition k , $\text{MGS}_{:,k}^{(n)} \in \mathbb{R}^V$ forms an individual-level brain activation map used for subsequent group inference and comparison with GLM-based activation.

For each subject n and vertex v , we computed a resting-state MGS map $\text{MGS}_{v,\text{rest}}^{(n)}$ from the subject’s resting-state fMRI by segmenting it into pseudo-blocks that match the motor-task block length (and number of blocks) and applying Eq.(2) with the same distance templates $\mathbf{d}_{G,k,b}$ in the same block order as the task data. No random sampling was used for selecting resting-state pseudo-blocks.

For each task condition k , we then formed a within-subject contrast

$$\Delta \text{MGS}_{v,k}^{(n)} = \text{MGS}_{v,k}^{(n)} - \text{MGS}_{v,\text{rest}}^{(n)}. \quad (3)$$

Group-level task modulation was assessed at each (v, k) by testing $H_0 : \Delta \text{MGS}_{v,k} = 0$ across subjects using a two-sided Wilcoxon signed-rank test (non-parametric, without assuming normality). Multiple comparisons across vertices were controlled separately for each condition using the Benjamini–Hochberg FDR [2] procedure, declaring significance at $p_{FDR} < 0.05$. Significant vertices were labeled as *activated* if $\text{median}_n(\Delta \text{MGS}_{v,k}^{(n)}) > 0$ and *suppressed* if $\text{median}_n(\Delta \text{MGS}_{v,k}^{(n)}) < 0$.

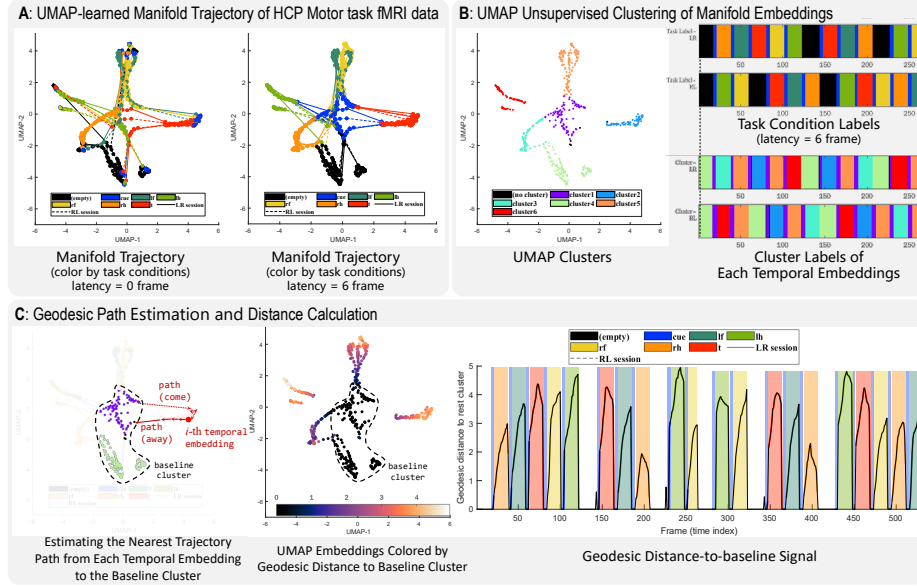


Fig. 2. Manifold trajectory, clustering, and geodesic distance to baseline. (A) Manifold trajectories of group-averaged HCP motor-task fMRI colored by condition labels (0-frame vs. 6-frame delay shift). (B) Clusters in the embedding (left) and the corresponding LR/RL temporal sequences of condition labels and cluster assignments (right). (C) Diagram of frame-wise geodesic path estimation (left), embeddings colored by distance-to-baseline (middle), and the resulting frame-wise geodesic distance signal over time (right; shaded bands denote task blocks).

3 Experimental Results

3.1 Manifold Trajectory Reveals Condition-aligned Petals and Geodesic Distance Signal

Figure 2 summarizes the group-level manifold geometry learned from HCP motor-task fMRI and motivates our distance-based analysis. Fig. 2A shows the 2D UMAP embedding of the group-averaged time series, where successive frames trace a petal-like looped trajectory. With nominal task timing (0-frame latency), points from the same condition are partially mixed across petals, consistent with hemodynamic delay; applying a 6-frame shift (~ 4.32 s) yields substantially more condition-coherent, petal-specific structure (Fig. 2A, right). Unless otherwise noted, all subsequent analyses use the 6-frame-shifted timing.

This structure also emerges without task labels: unsupervised DBM clustering produces compact, well-separated manifold clusters (Fig. 2B) that closely correspond to the shifted condition labels over time. Using two central clusters as a baseline region, we compute for each point the shortest along-trajectory path length back to baseline, producing a geodesic distance-to-baseline signal

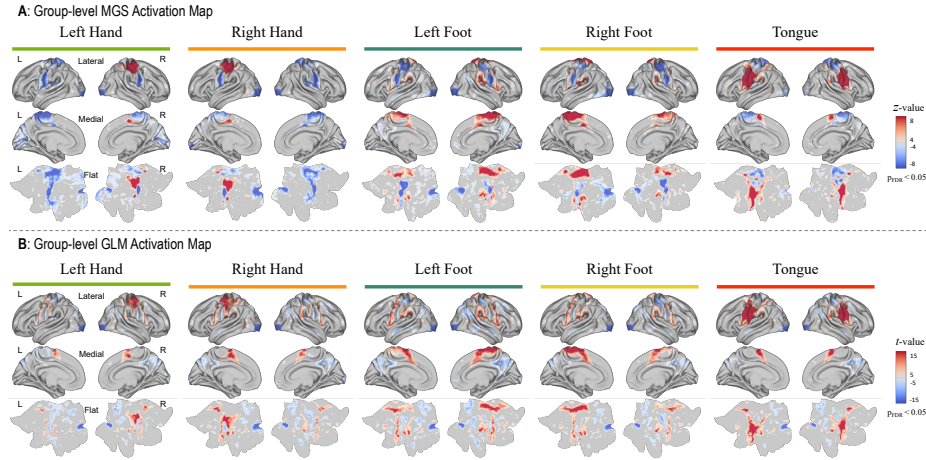


Fig. 4. Group-level activation detection maps for motor task: MGS vs. GLM. (A) MGS group maps (signed z -values; $p_{FDR} < 0.05$) and (B) GLM group maps (t -values; $p_{FDR} < 0.05$) for left/right hand, left/right foot, and tongue conditions.

3.3 Group-level MGS Activation Detection and Comparison with GLM

Figure 4 contrasts group-level activation maps obtained with the proposed MGS framework versus a conventional GLM analysis for each motor effector. Across conditions, both approaches recover the canonical motor network organization: strong, contralateral sensorimotor effects for left/right hand around the precentral–postcentral area, medial wall engagement for foot movements near the paracentral lobule, and ventral sensorimotor/peri-opercular involvement for tongue. These results indicate that MGS preserves the expected task-specific somatotopy at the group-level.

Notably, the MGS maps (Fig. 4A) are more spatially compact and condition-selective, showing less diffuse effects outside primary sensorimotor regions than the GLM maps (Fig. 4B), which may reflect shared variance across blocks and sensitivity to HRF specification. Moreover, MGS provides a signed task–rest modulation (activation vs. suppression), whereas GLM inference is tied to a fixed HRF-convolved regressor and in practice often emphasizes positive responses under common modeling choices. Together, these properties suggest that MGS can yield sharper, more discriminative group-level maps by exploiting an intrinsic, manifold-derived distance-to-baseline reference rather than relying solely on an explicit HRF model.

4 Conclusion

In this work, we presented a manifold-geometric framework for task fMRI activation mapping. On HCP motor-task data, UMAP embeddings reveal robust petal-

like looped trajectories, where delay-shifted labels improve condition alignment and expose BOLD latency, and identify condition-specific clusters together with a central baseline cluster as an intrinsic baseline. We quantify task engagement by geodesic distance-to-baseline along the trajectory graph and introduce Manifold Geodesic-based Similarity (MGS) to translate these geometric excursions into vertex-wise activation estimates for both individual- and group-level analyses. MGS recovers canonical motor somatotopy while producing more compact and condition-selective maps than GLM, and it additionally highlights task-relevant effects that can be weak under HRF-constrained linear modeling. Limitations include evaluation on a single paradigm and reliance on a group-level manifold. Future work will extend to additional tasks and develop subject-adaptive manifolds and geodesic statistics.

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